Published by Institute of Physics Publishing for SISSA/ISAS
Received: *March 6, 2002*
Revised: *April 11, 2002*
Accepted: *May 28, 2002*

D-brane bound states from charged macroscopic strings

Alok Kumar, Sudipta Mukherji and Kamal L. Panigrahi

Institute of Physics
Bhubaneswar 751 005, India
E-mail: kumar@iopb.res.in, mukherji@iopb.res.in, kamal@iopb.res.in

Abstract: We construct new D-brane bound states using charged macroscopic type-IIB string solutions. A generic bound state solution, when dimensionally reduced, carries multiple gauge charges. Starting with $D = 9$ charged macroscopic strings, we obtain solutions in $D = 10$, which are interpreted as carrying (F, $D0$, $D2$) charges as well as nonzero momenta. The masses and charges are also explicitly shown to satisfy the non-threshold bound of 1/2 BPS objects. Our solutions reduce to the known D-brane bound state solutions with appropriate restrictions in the parameter space. We further generalize the results to $(Dp - D(p + 2))$ bound state in IIA/B theories, giving an explicit example with $p = 1$.

Keywords: Superstrings and Heterotic Strings, D-branes, String Duality.

JHEP05(2002)050

Contents

1. Introduction

Bound states of D-branes [1]–[8] have been an interesting area of investigation due to their applications in understanding the non- perturbative aspects of various string and gauge theories [9, 10, 11]. In particular, the supergravity configurations of supersymmetric D-branes and their bound states were used extensively in testing various conjectures which involve knowledge of string theory or gauge theory beyond their perturbative regime. In view of their widespread applications, it is often very useful to generalize D-brane bound state constructions. As a step in this direction, in the present paper, we first give a construction of generalized $D0-D2$ bound state using charged macroscopic strings [12]. These strings carry, in general, many parameters associated with the charges and currents. As a consequence, the D-brane bound states have many nontrivial charges. We argue that these configurations can also in general carry F-string charges and momenta along their worldvolume directions. We then further extend our result by presenting the generalized $(Dp - D(p+2))$ bound states. In this context we give an explicit example for the $D1 - D3$ case.

A method for obtaining the bound state of various D-branes is described explicitly in [13]. This is done by smearing the brane of type-IIA (IIB) theory along certain transverse direction and mixing it with a longitudinal direction through a coordinate transformation to construct 'tilted' brane in $D = 10$. Finally, an application of T-duality on the configuration leads to a bound state solution in the dualized IIB (IIA) theory. As stated earlier, in this paper we concentrate mainly on $D0 - D2$ bound state of the IIA theory. They are constructed by starting with a D-string solution in the IIB theory in $D = 10$, which is also delocalized along one of the transverse directions. These delocalized solutions can also be obtained by decompactifying a fundamental string solution in $D = 9$ to $D = 10$. Further, one applies an S- duality transformation in $D = 10$ to construct $\mathrm{SL}(2,\mathbb{Z})$ multiplets. One, however, knows the existence of more general string solutions in heterotic as well as type-II strings in all dimensions $4 \leq D \leq 9$ [12, 14], which carry vectorial charges and currents. We make use of such charged macroscopic string solutions to generate a generalized smeared (delocalized) D-string in $D = 10$. We then apply the procedure for obtaining the D-brane bound states, as described above, on such delocalized configurations. For the special case when vectorial charges are set to zero, our delocalized solutions reduce to the $D = 10$ smeared string solutions of [13]. We therefore have a generalization of the $D0 - D2$ bound state by starting with the charge macroscopic string solutions in $D = 9$.

We also show that by smearing some of the transverse directions of our 'generalized' $D0 - D2$ bound state discussed above, and applying T-duality along these additional directions, we can construct new $Dp - D(p + 2)$ bound states as well. We work out the case, $p = 1$, explicitly. In fact even more general bound states can be constructed by using $D < 9$ charged macroscopic string solutions. For the purpose of this paper, we however mainly restrict ourselves to the $D = 9$ solutions. We also explicitly verify the (non-threshold) BPS condition in all the examples discussed in the paper.

The charged string solutions are generated from the neutral ones by a solution generating transformation. They are parametrized by a group $O(d - 1, 1; d - 1, 1)$, where d is the number of compactified directions. In particular, the solution described in [12, 14] is parametrized by two nontrivial parameters of the transformation namely α and β. The general solution for arbitrary values of α and β for $D < 10$ is given in [14]. Explicit supersymmetry property of these solutions are given for $\beta = 0$, $\alpha \neq 0$ and $\alpha = -\beta$ in [14]. For algebraic simplifications, in this paper, we will deal with these choices of α and β. Decompactifications, $\mathrm{SL}(2,\mathbb{Z})$ transformations and T- duality operations for general α and β are possible to write down, but algebraically more complicated. The rest of the paper is organized as follows. In section 2, we present a review of the charged macroscopic string solutions. In section 3, we first review the construction of $(D0-D2)$ bound state in $D = 10$, starting from D-string solutions. We then discuss the construction of $Dp-D(p+2)$ bound states from these ones by application of T-duality along other (smeared) transverse directions. Generalization of these non-threshold bound states, replacing the neutral strings by charged macroscopic strings is presented for the specific solution $\alpha = -\beta$ in section 4. Results for $\beta = 0, \alpha \neq 0$ solution are presented in section 5. Here we also present further generalization of the results by using (p, q) type-IIB strings and show the existence of the non-threshold bound states for any (p, q)-string, generated from a D- string via $\mathrm{SL}(2,\mathbb{Z})$.

JHEP05(2002)050

We explicitly calculate the ADM mass and verify the mass and charge relationship in these examples to check BPS nature of these configurations. Conclusions and discussions are presented in section 6.

2. Charged macroscopic strings

2.1 Charged macroscopic string solution

We start by writing down the bosonic backgrounds associated with the charged macroscopic strings in space-time dimensions D for type-II theories. For charged string solutions below, we use notations and conventions that are identical to the ones in [15] and [14]. The solution is given by,

$$\begin{aligned}
ds^2 &= r^{D-4}\Delta^{-1}\Big[-\left(r^{D-4}+C\right)dt^2 + C(\cosh\alpha-\cosh\beta)dt dx^{D-1} + \\
&\qquad + \left(r^{D-4}+C\cosh\alpha\cosh\beta\right)\left(dx^{D-1}\right)^2\Big] + \\
&\quad + \left(dr^2 + r^2 d\Omega^2_{D-3}\right), && (2.1)\\
B_{(D-1)t} &= \frac{C}{2\Delta}(\cosh\alpha+\cosh\beta)\left\{r^{D-4}+\frac{1}{2}C(1+\cosh\alpha\cosh\beta)\right\}, && (2.2)\\
e^{-\Phi} &= \frac{\Delta^{1/2}}{r^{D-4}}, && (2.3)\\
A^{(a)}_t &= -\frac{n^{(a)}}{2\sqrt{2}\Delta}C\sinh\alpha\left\{r^{D-4}\cosh\beta+\frac{1}{2}C(\cosh\alpha+\cosh\beta)\right\}\\
&\quad \text{for} \quad 1\le a\le(10-D),\\
&= -\frac{p^{(a-10+D)}}{2\sqrt{2}\Delta}C\sinh\beta\left\{r^{D-4}\cosh\alpha+\frac{1}{2}C(\cosh\alpha+\cosh\beta)\right\}\\
&\quad \text{for} \quad (10-D)+1\le a\le(20-2D), && (2.4)\\
A^{(a)}_{D-1} &= -\frac{n^{(a)}}{2\sqrt{2}\Delta}C\sinh\alpha\left\{r^{D-4}+\frac{1}{2}C\cosh\beta(\cosh\alpha+\cosh\beta)\right\}\\
&\quad \text{for} \quad 1\le a\le(10-D),\\
&= \frac{p^{(a-10+D)}}{2\sqrt{2}\Delta}C\sinh\beta\left\{r^{D-4}+\frac{1}{2}C\cosh\alpha(\cosh\alpha+\cosh\beta)\right\}\\
&\quad \text{for} \quad (10-D)+1\le a\le(20-2D), && (2.5)\\
M_D &= I_{20-2D} + \begin{pmatrix} Pnn^T & Qnp^T \\ Qpn^T & Ppp^T \end{pmatrix}, && (2.6)
\end{aligned}$$

where,

$$\begin{aligned}
\Delta &= r^{2(D-4)} + Cr^{D-4}(1+\cosh\alpha\cosh\beta) + \frac{C^2}{4}(\cosh\alpha+\cosh\beta)^2, && (2.7)\\
P &= \frac{C^2}{2\Delta}\sinh^2\alpha\sinh^2\beta, && (2.8)\\
Q &= -C\Delta^{-1}\sinh\alpha\sinh\beta\left\{r^{D-4}+\frac{1}{2}C(1+\cosh\alpha\cosh\beta)\right\}. && (2.9)
\end{aligned}$$

with $n^{(a)}$, $p^{(a)}$ being the components of $(10-D)$-dimensional unit vectors. A_μ's in eqs. (2.4), (2.5) are the gauge fields appearing due to the Kaluza-Klein (KK) reductions of the ten dimensional metric and the 2-form antisymmetric tensor coming from the NS-NS sector, $B_{\mu\nu}$ is the NS-NS 2-form field and Φ is the dilaton in the D-dimensional space-time. In the above configuration, C is a constant related to tension of the string. The matrix M_D parameterizes the moduli fields. The exact form of this parameterization depends on the form of the $O(10-D, 10-D)$ metric used. The above solution has been written for a diagonal metric of the form:

$$L_D = \begin{pmatrix} -I_{10-D} & \\ & I_{10-D} \end{pmatrix}. \tag{2.10}$$

One sometimes also uses an off-diagonal metric convention as in eq. (2.17) below:

$$L = \begin{pmatrix} & I_{10-D} \\ I_{10-D} & \end{pmatrix}. \tag{2.11}$$

These two conventions are however related by:

$$L_D = \hat{P} L \hat{P}^T\,, \qquad M_D = \hat{P} M \hat{P}^T\,, \tag{2.12}$$

where

$$\hat{P} = \frac{1}{\sqrt{2}} \begin{pmatrix} -I_{10-D} & I_{10-D} \\ I_{10-D} & I_{10-D} \end{pmatrix}. \tag{2.13}$$

The gauge fields in two conventions are related as:

$$\begin{pmatrix} A^1_\mu \\ A^2_\mu \end{pmatrix} = \hat{P} \begin{pmatrix} \hat{A}^1_\mu \\ \hat{A}^2_\mu \end{pmatrix}, \tag{2.14}$$

with $A^{1,2}_\mu$'s in the above equation being $(10-D)$-dimensional columns consisting of the gauge fields A_μ's defined in (2.4), (2.5), and coming from the left and the right-moving sectors of string theory.

2.2 Decompactified solutions for $\beta = 0$, $\alpha \neq 0$ and $\alpha = -\beta$

We now discuss the decompactification of the D-dimensional solutions (2.1)–(2.9) to $D = 10$. As stated earlier, we now restrict ourselves to specific values of the parameters: $\alpha = -\beta$ and $\beta = 0, \alpha \neq 0$ for algebraic simplifications. These special cases, in particular the later possibility, encompasses all the nontrivialities of our construction. We also restrict ourselves to $D = 9$ for constructing $D0-D2$ bound state. First we start with the seed solution (with $\beta = 0$, α arbitrary) given as:

$$\begin{aligned} ds^2 &= \frac{1}{\cosh^2\frac{\alpha}{2} e^{-E} - \sinh^2\frac{\alpha}{2}} \left(-dt^2 + \left(dx^{D-1}\right)^2\right) + \\ &\quad + \frac{\sinh^2\frac{\alpha}{2}\left(e^{-E}-1\right)}{\left(\cosh^2\frac{\alpha}{2} e^{-E} - \sinh^2\frac{\alpha}{2}\right)^2} \left(dt + dx^{D-1}\right)^2 + \sum_{i=1}^{D-2} dx^i dx^i\,, \\ B_{(D-1)t} &= \frac{\cosh^2\frac{\alpha}{2}\left(e^{-E}-1\right)}{\cosh^2\frac{\alpha}{2} e^{-E} - \sinh^2\frac{\alpha}{2}}\,, \end{aligned}$$

$$
\begin{aligned}
A^{(1)}_{D-1} &= A^{(1)}_t = -\frac{1}{2\sqrt{2}} \times \frac{\sinh\alpha(e^{-E}-1)}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}\,,\\
\Phi &= -\ln\left(\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}\right),
\end{aligned}
\tag{2.15}
$$

with e^{-E} being the Green function in the 5-dimensional transverse space:

$$
e^{-E} = \left(1 + \frac{C}{r^5}\right),
\tag{2.16}
$$

and constant C determines the string tension.

Now, to construct delocalized solutions in $D = 10$, we decompactify the above solution back to ten dimensions. The decompactification exercise is done following a set of notations given in [16]. When restricted to the NS-NS sector of type-II theories, they can be written as:

$$
\begin{aligned}
&\hat{G}_{ab} = G^{(10)}_{[a+(D-1),b+(D-1)]}\,, \qquad \hat{B}_{ab} = B^{(10)}_{[a+(D-1),b+(D-1)]}\,,\\
&\hat{A}^{(a)}_{\bar{\mu}} = \frac{1}{2}\hat{G}^{ab}G^{(10)}_{[b+(D-1),\bar{\mu}]}\,,\\
&\hat{A}^{(a+(10-D))}_{\bar{\mu}} = \frac{1}{2}B^{(10)}_{[a+(D-1),\bar{\mu}]} - \hat{B}_{ab}A^{(b)}_{\bar{\mu}}\,,\\
&G_{\bar{\mu}\bar{\nu}} = G^{(10)}_{\bar{\mu}\bar{\nu}} - G^{(10)}_{[(a+(D-1)),\bar{\mu}]}G^{(10)}_{[(b+(D-1)),\bar{\nu}]}\hat{G}^{ab}\,,\\
&B_{\bar{\mu}\bar{\nu}} = B^{(10)}_{\bar{\mu}\bar{\nu}} - 4\hat{B}_{ab}A^{(a)}_{\bar{\mu}}A^{(b)}_{\bar{\nu}} - 2\left(A^{(a)}_{\bar{\mu}}A^{(a+(10-D))}_{\bar{\nu}} - A^{(a)}_{\bar{\nu}}A^{(a+(10-D))}_{\bar{\mu}}\right),\\
&\Phi = \Phi^{(10)} - \frac{1}{2}\ln\det\hat{G}\,, \qquad 1 \le a,b \le 10-D\,, \qquad 0 \le \bar{\mu},\bar{\nu} \le (D-1)\,.
\end{aligned}
\tag{2.17}
$$

We now start with the nine-dimensional ($D = 9$) solution in (2.15) and following the Kaluza-Klein (KK) compactification mechanism summarized above, write down the solution directly in ten dimensions [14]. For $\beta = 0$, only nonzero background fields are then given by

$$
\begin{aligned}
ds^2 &= \frac{1}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}\left(-dt^2 + \left(dx^8\right)^2\right) +\\
&\quad + \frac{\sinh^2\frac{\alpha}{2}\left(e^{-E}-1\right)}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}\left(dt + dx^8\right)^2 +\\
&\quad + \frac{\sinh\alpha\left(e^{-E}-1\right)}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}dx^9\left(dt + dx^8\right) + \sum_{i=1}^{7}dx^i dx^i + \left(dx^9\right)^2,
\end{aligned}
\tag{2.18}
$$

$$
\begin{aligned}
B_{8t} &= \frac{\cosh^2\frac{\alpha}{2}\left(e^{-E}-1\right)}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}\,,\\
B_{9t} &= -\frac{\sinh\alpha}{2}\frac{\left(e^{-E}-1\right)}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}} = B_{98}\,.
\end{aligned}
\tag{2.19}
$$

The dilaton in ten dimensions remains same as the one in (2.15):

$$
\Phi^{(10)} = -\ln\left(\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}\right).
\tag{2.20}
$$

For $\alpha = -\beta$, on the other hand, we have $D = 9$ solutions given by a metric:

$$ds^2 = -\frac{1}{1+\frac{C\cosh^2\alpha}{r^5}}dt^2 + \frac{1}{1+\frac{C}{r^5}}\left(dx^8\right)^2 + \sum_{i=1}^{7} dx^i dx^i. \tag{2.21}$$

The only non-zero component of the antisymmetric tensor is of the form

$$B_{t8} = -\frac{C\cosh\alpha}{2}\left[\frac{1}{(r^5+C)} + \frac{1}{(r^5+C\cosh^2\alpha)}\right]. \tag{2.22}$$

We also have a nontrivial modulus parameterizing the $O(1,1)$ matrix M_D in eq. (2.6):

$$\hat{G}_{99} \equiv \hat{g} = \frac{1+\frac{C\cosh^2\alpha}{r^5}}{1+\frac{C}{r^5}}. \tag{2.23}$$

The two gauge fields appearing in equations (2.17) for $D = 9$ are of the form:

$$\hat{A}^1_t = \frac{C\sinh\alpha\cosh\alpha}{2(r^5+C\cosh^2\alpha)}, \qquad \hat{A}^1_8 = 0, \tag{2.24}$$

$$\hat{A}^2_t = 0, \qquad \hat{A}^2_8 = \frac{-C\sinh\alpha}{2(r^5+C)}. \tag{2.25}$$

The decompactified solution for $\alpha = -\beta$ case in $D = 10$ is given by:

$$\begin{aligned} ds^2 = &-\frac{\left(1-\frac{C}{r^5}\sinh^2\alpha\right)}{1+\frac{C}{r^5}}(dt)^2 + \frac{1+\frac{C}{r^5}\cosh^2\alpha}{1+\frac{C}{r^5}}\left(dx^9\right)^2 + \\ &+\frac{\frac{2C}{r^5}\cosh\alpha\sinh\alpha}{1+\frac{C}{r^5}}dx^9 dt + \frac{1}{1+\frac{C}{r^5}}\left(dx^8\right)^2 + \sum_{i=1}^{7}\left(dx^i\right)^2, \end{aligned} \tag{2.26}$$

antisymmetric NS-NS $B_{\mu\nu}$:

$$B_{98} = -C\frac{\sinh\alpha}{r^5+C}, \qquad B_{t8} = -C\frac{\cosh\alpha}{r^5+C}, \tag{2.27}$$

and by the dilaton:

$$\Phi^{(10)} = -\ln\left(1+\frac{C}{r^5}\right). \tag{2.28}$$

Before applying $\mathrm{SL}(2,\mathbb{Z})$ transformation to the solutions in eqs. (2.18)–(2.20) and (2.26)–(2.28), we now describe the general construction of $Dp - D(p+2)$ bound states starting from D- strings of type-IIB [13].

3. Construction of $Dp - D(p+2)$ bound states

In this section we first review the construction of $D0 - D2$ non-threshold bound states in type-IIA string theories from the (neutral) D-strings in type-IIB. Our starting point in this case is the D-string solution, smeared (delocalized) along a transverse direction $x \equiv x^9$.

This solution is given as:

$$
\begin{aligned}
ds^2 &= \sqrt{H}\left(\frac{-dt^2+dy^2}{H}+dx^2+\sum_{i=1}^{7}\left(dx^i\right)^2\right),\\
A^{(2)} &= \pm\left(\frac{1}{H}-1\right)dt\wedge dy\,,\\
e^{\Phi_b^{(10)}} &= H\,. &(3.1)\\
H &= 1+\frac{C}{r^5}\,. &(3.2)
\end{aligned}
$$

is the solution of the Greens' function equation:

$$
\Box H = CA_{N-1}\prod_{i=1}^{N}\delta\left(x^i\right), \tag{3.3}
$$

where N indicates the transverse directions which are not smeared and A_N is the area of S^N orthogonal to the brane. To compare with the solutions in section 2, one identifies $H = e^{-E}$. Then a rotation is performed between x and y directions:

$$
\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} \tag{3.4}
$$

to mix the longitudinal and transverse coordinates of the above solution. The solution in eq. (3.1) then transforms to:

$$
\begin{aligned}
ds^2 &= \sqrt{H}\Bigg[\frac{-dt^2}{H}+\left(\frac{\cos^2\phi}{H}+\sin^2\phi\right)d\tilde{y}^2+\left(\frac{\sin^2\phi}{H}+\cos^2\phi\right)d\tilde{x}^2+\\
&\qquad +2\cos\phi\sin\phi\left(\frac{1}{H}-1\right)d\tilde{y}d\tilde{x}+\sum_{i=2}^{8}\left(dx^i\right)^2\Bigg],\\
A^{(2)} &= \pm\left(\frac{1}{H}-1\right)dt\wedge\left(\cos\phi d\tilde{y}+\sin\phi d\tilde{x}\right),\\
e^{\Phi_b^{(10)}} &= H\,. &(3.5)
\end{aligned}
$$

Finally, a T-duality transformation [17, 13] on coordinate $\tilde{x}$ gives the following classical solution in the type-IIA theory:

$$
\begin{aligned}
ds^2 &= \sqrt{H}\left(-\frac{dt^2}{H}+\frac{d\tilde{x}^2+d\tilde{y}^2}{1+(H-1)\cos\phi}+\sum_{i=2}^{8}(dx^i)^2\right),\\
A^{(1)} &= \pm\left(\frac{1}{H}-1\right)\sin\phi dt\,,\\
A^{(3)} &= \pm\frac{(H-1)\cos\phi}{1+(H-1)\cos^2\phi}dt\wedge d\tilde{x}\wedge d\tilde{y}\,,\\
B^{(a)} &= \frac{(H-1)\cos\phi\sin\phi}{1+(H-1)\cos^2\phi}d\tilde{x}\wedge d\tilde{y}\,,\\
e^{\Phi_a^{(10)}} &= \frac{H^{\frac{3}{2}}}{1+(H-1)\cos^2\phi}\,. &(3.6)
\end{aligned}
$$

Solution in eq. (3.6) can be interpreted as a $D0-D2$ bound state. The $D0$ and $D2$ charge densities carried by the above bound state solution are given as:

$$
\begin{aligned}
Q_0 &= 5C\sin\phi A_6 \,, \\
Q_2 &= 5C\cos\phi A_6 \,,
\end{aligned}
\tag{3.7}
$$

where A_6 is the area of S^6 orthogonal to the brane. The ADM mass [18, 13] is defined by the expression:

$$
m = \int \sum_{i=1}^{9-p} n^i \left[\sum_{j=1}^{9-p} (\partial_j h_{ij} - \partial_i h_{jj}) - \sum_{a=1}^{p} \partial_i h_{aa} \right] r^{8-p} d\Omega \,, \tag{3.8}
$$

where n^i is a radial unit vector in the transverse space and $h_{\mu\nu}$ is the deformation of the Einstein-frame metric from flat space in the asymptotic region. We also should mention here that in order to write mass and charges, we have tuned the gravitational constant to a suitable value. The ADM mass density in the present context is found to be:

$$
m_{0,2} = 5CA_6 \,. \tag{3.9}
$$

Mass and charge densities given in eqs. (3.7) and (3.9) above satisfy the BPS condition:

$$
(m_{0,2})^2 = \left(Q_0^2 + Q_2^2\right) . \tag{3.10}
$$

To generalize the results to other D-brane bound states, we smear one more transverse direction, $x^i \sim x^7$ in eq. (3.6). Finally applying T-duality along this direction, one is able to construct a $D1-D3$ bound state. In the dualized (IIB) theory, one can write down all the field components trivially. One finds an exact matching with the $D1-D3$ solution of [13]. In particular, matching of the 4-form field can be seen by using the identity:

$$
\begin{aligned}
A^{(4)} &\equiv \frac{(H-1)\cos\phi}{1+(H-1)\cos^2\phi} - \frac{1}{2}\frac{(H-1)}{H}\frac{\sin\phi(H-1)\cos\phi\sin\phi}{1+(H-1)\cos^2\phi} \\
&= \frac{(H-1)\cos\phi}{2H}\left[1+\frac{H}{1+(H-1)\cos^2\phi}\right] .
\end{aligned}
\tag{3.11}
$$

We give the final $D1-D3$ solution [13] for completeness as well as for later use:

$$
\begin{aligned}
ds^2 &= \sqrt{H}\Bigg\{ \frac{-dt^2+(dy^2)^2}{H} + \frac{d\tilde{y}^2+d\tilde{x}^2}{1+(H-1)\cos^2\phi} + dr^2 + \\
&\qquad + r^2\left(d\theta^2+\sin^2\theta\left(d\phi_1^2+\sin^2\phi_1\left(d\phi_2^2+\sin^2\phi_2\left(d\phi_3^2+\sin^2\phi_3 d\phi_4^2\right)\right)\right)\right)\Bigg\} \\
A^{(4)} &= \mp\frac{\cos\phi}{2}\frac{H-1}{H}\left(1+\frac{H}{1+(H-1)\cos^2\phi}\right)\times dt\wedge d\tilde{y}\wedge dy^2\wedge d\tilde{x} \pm \\
&\qquad \pm 4C\cos\phi\,\sin^4\theta\sin^3\phi_1\sin^2\phi_2\cos\phi_3 d\phi\wedge d\phi_1\wedge d\phi_2\wedge d\phi_4 \,, \\
A^{(2)} &= \pm\frac{H-1}{H}\sin\phi\,dt\wedge dy^2 \,, \\
B^{(b)} &= \frac{(H-1)\cos\phi\sin\phi}{1+(H-1)\cos^2\phi} d\tilde{x}\wedge d\tilde{y} \,, \\
e^{\Phi_b^{(10)}} &= \frac{H}{1+(H-1)\cos^2\phi} \,,
\end{aligned}
\tag{3.12}
$$

where $H = 1 + C/r^4$.

One can now repeat this process to generate all the bound states of $Dp-D(p+2)$ type in a similar manner. We now apply the general procedure described above to the charged macroscopic string solutions of section 2.

4. Generalized $(Dp - D(p+2))$ bound state

In this section we construct non-threshold bound states which are the generalization of the $D0-D2$ bound state presented in the previous section. Here we use the $\alpha = -\beta$ solutions of section 2 and postpone the discussion of $\beta = 0, \alpha \neq 0$ solutions to section 5.

4.1 Generalization of $D0 - D2$ bound states

The delocalized elementary string solution is given in eqs. (2.26)–(2.28). A delocalized D-string in $D = 10$ can be generated from this solution by an application of S-duality transformation [19] which transforms an elementary string into a D-string. The metric, antisymmetric 2-form ($B_{\mu\nu}$) and the dilaton for the delocalized D-string solution are given by:

$$\begin{aligned} ds^2 = & -\frac{1-\frac{C}{r^5}\sinh^2\alpha}{\sqrt{1+\frac{C}{r^5}}}(dt)^2 + \frac{1+\frac{C}{r^5}\cosh^2\alpha}{\sqrt{1+\frac{C}{r^5}}}\left(dx^9\right)^2 + \\ & + \frac{2\frac{C}{r^5}\sinh\alpha\cosh\alpha}{\sqrt{1+\frac{C}{r^5}}}dx^9 dt + \frac{1}{\sqrt{1+\frac{C}{r^5}}}\left(dx^8\right)^2 + \\ & + \sqrt{1+\frac{C}{r^5}}\sum_{i=1}^{7}\left(dx^i\right)^2, \end{aligned} \tag{4.1}$$

$$B^{(2)}_{98} = B_{98}\,, \qquad B^{(2)}_{t8} = B_{t8}\,, \qquad e^{\Phi_b^{(10)}} = 1 + \frac{C}{r^5}\,, \tag{4.2}$$

with B_{98} and B_{t8} as given in eq. (2.27) and the superscript on B denotes the R-R nature of the 2-form antisymmetric tensors. The next step of our construction is to apply rotation in $(x^9 - x^8)$-plane by an angle ϕ which gives the following configuration:

$$\begin{aligned} ds^2 = & -\frac{1-\frac{C}{r^5}\sinh^2\alpha}{\sqrt{1+\frac{C}{r^5}}}dt^2 + \frac{1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi}{\sqrt{1+\frac{C}{r^5}}}\left(d\tilde{x}^9\right)^2 + \\ & + \frac{1+\frac{C}{r^5}\cosh^2\alpha\sin^2\phi}{\sqrt{1+\frac{C}{r^5}}}\left(d\tilde{x}^8\right)^2 + \frac{2\frac{C}{r^5}\cosh\alpha\sinh\alpha\cos\phi}{\sqrt{1+\frac{C}{r^5}}}d\tilde{x}^9 dt - \\ & - \frac{2\frac{C}{r^5}\cosh\alpha\sinh\alpha\sin\phi}{\sqrt{1+\frac{C}{r^5}}}d\tilde{x}^8 dt - \frac{2\frac{C}{r^5}\cosh^2\alpha\sin\phi\cos\phi}{\sqrt{1+\frac{C}{r^5}}}\left(d\tilde{x}^9\right)d\tilde{x}^8 + \\ & + \sqrt{1+\frac{C}{r^5}}\sum_{i=1}^{7}\left(dx^i\right)^2, \end{aligned} \tag{4.3}$$

$$
\begin{aligned}
B^{(2)}_{\tilde{8}t} &= \frac{C}{C+r^5}\cosh\alpha\cos\phi\,,\\
B^{(2)}_{\tilde{9}t} &= \frac{C}{C+r^5}\cosh\alpha\sin\phi\,,\\
B^{(2)}_{\tilde{9}\tilde{8}} &= \frac{-C}{C+r^5}\sinh\alpha\,.
\end{aligned}
\tag{4.4}
$$

Finally we apply T-duality along the $\tilde{x}^9$-direction. By following the prescription as given in [13, 17], we end up with the structure for the metric, NS-NS $B_{\mu\nu}$, as well as $1-$ form and $3-$ form fields of type-IIA theory:

$$
\begin{aligned}
d\mathcal{S}^2 = &-\frac{1+\frac{C}{r^5}(1-\cosh^2\alpha\sin^2\phi)}{\sqrt{1+\frac{C}{r^5}}\left(1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi\right)}dt^2+\\
&+\frac{\sqrt{1+\frac{C}{r^5}}}{1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi}\left(d\tilde{x}^9\right)^2+\frac{1+\frac{C}{r^5}\cosh^2\alpha}{\sqrt{1+\frac{C}{r^5}}\left(1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi\right)}\left(d\tilde{x}^8\right)^2-\\
&-\frac{\frac{C}{r^5}\sinh\alpha\cosh\alpha\sin\phi}{\sqrt{1+\frac{C}{r^5}}\left(1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi\right)}dtd\tilde{x}^8+\sqrt{1+\frac{C}{r^5}}\sum_{i=1}^{7}\left(dx^i\right)^2,
\end{aligned}
\tag{4.5}
$$

$$
\begin{aligned}
e^{\Phi_b^{(10)}} &= \frac{(1+\frac{C}{r^5})^{3/2}}{1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi},\\
\mathcal{A}_t &= \frac{C}{C+r^5}\cosh\alpha\sin\phi\,, \qquad \mathcal{A}_{\tilde{x}^8} = \frac{-C}{C+r^5}\sinh\alpha\\
\mathcal{B}_{\tilde{9}t} &= \frac{\frac{-C}{r^5}\sinh\alpha\cosh\alpha\cos\phi}{1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi}\,, \qquad \mathcal{B}_{\tilde{9}\tilde{8}} = \frac{\frac{C}{r^5}\sin\phi\cos\phi\cosh^2\alpha}{1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi}\,,\\
\mathcal{A}_{\tilde{9}t\tilde{8}} &= \frac{-\frac{C}{r^5}\cosh\alpha\cos\phi}{1+\frac{C}{r^5}\cosh^2\alpha\cos^2\phi}\,.
\end{aligned}
\tag{4.6}
$$

This solution is a generalization of the $D0-D2$ bound state, where in addition we have turned on NS-NS 2-form as well. To show that this indeed represents a non-threshold $1/2$ BPS state, in the next sub-section, we explicitly verify the mass-charge relation of these bound states.

4.2 Mass-charge relationship

To show the BPS nature of our solution, we perform a dimensional reduction of our solution along $\tilde{x}^8$ and $\tilde{x}^9$ directions. By dimensional reduction, one avoids any ambiguity that may arise due to the presence of purely spatial components of the p-form fields in equation (4.5), (4.6) above. As a result, all the nonzero charges in our case arise from the temporal part of 1-form field components only, in this eight dimensional theory [20]. They are $A^1_t \sim \mathcal{A}_t$, $A^2_t \sim \mathcal{B}_{\tilde{9}t}$, $A^3_t \sim \mathcal{A}_{\tilde{9}t\tilde{8}}$ from the components of the p-form fields, as well as off-diagonal component $g_{t\tilde{8}}/g_{\tilde{8}\tilde{8}}$ of the metric. The charges associated with these field strengths and metric components can be read from the solutions (4.5), (4.6). They are:

$$
\begin{aligned}
Q_1 &= 5C\cosh\alpha\sin\phi\,,\\
Q_2 &= -5C\sinh\alpha\cosh\alpha\cos\phi\,,\\
Q_3 &= -5C\cosh\alpha\cos\phi\,,\\
P &= 5C\sinh\alpha\cosh\alpha\sin\phi\,,
\end{aligned}
\tag{4.7}
$$

where Q_i's are the charges corresponding to the field strengths of A_t^i that we just defined and P can be interpreted as the momentum along $\tilde{x}^8$ direction in the ten-dimensional theory. The mass-density of our bound state can be computed using (3.8) and is given by

$$
m_{(0,2)} = 5C\cosh^2\alpha\,. \tag{4.8}
$$

Comparing (4.7) and (4.8), we get the standard BPS condition as:

$$
(m_{0,2})^2 = {Q^2}_1 + {Q^2}_2 + {Q^2}_3 + P^2\,. \tag{4.9}
$$

This, in turn, implies the supersymmetric nature of the bound state.

4.3 Further generalization to $Dp - D(p+2)$

We now obtain a generalization of the $D1-D3$ bound state solution presented in eq. (3.12) by applying T-duality along x^7 direction on the generalized $D0-D2$ solution (4.5), (4.6) presented earlier in this section. The final result is given by:

$$
\begin{aligned}
d\mathcal{S}^2 &= -\frac{1+\frac{C}{r^4}(1-\cosh^2\alpha\sin^2\phi)}{\sqrt{1+\frac{C}{r^4}}(1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)}dt^2 + \frac{\sqrt{1+\frac{C}{r^4}}}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi}(d\tilde{x}^9)^2 + \\
&\quad + \frac{1+\frac{C}{r^4}\cosh^2\alpha}{\sqrt{1+\frac{C}{r^4}}(1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)}(d\tilde{x}^8)^2 + \frac{1}{\sqrt{1+\frac{C}{r^4}}}(dx^7)^2 - \\
&\quad - \frac{\frac{C}{r^4}\sinh\alpha\cosh\alpha\sin\phi}{\sqrt{1+\frac{C}{r^4}}(1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi)}dt d\tilde{x}^8 + \sqrt{1+\frac{C}{r^4}}\sum_{i=1}^{6}(dx^i)^2\,,\\
\mathcal{A}^{(2)}_{7t} &= \frac{C\sin\phi\cosh\alpha}{(r^4+C)}\,, \qquad \mathcal{A}^{(2)}_{7\tilde{8}} = -\frac{C\sinh\alpha}{(r^4+C)}\,,\\
\mathcal{A}^{(4)}_{\tilde{9}t\tilde{8}7} &= -\frac{(\frac{C}{r^4}\cosh\alpha\cos\phi)}{2(1+\frac{C}{r^4})}\left[1+\frac{1+\frac{C}{r^4}}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi}\right],\\
\mathcal{B}_{\tilde{9}t} &= \frac{\frac{-C}{r^4}\sinh\alpha\cosh\alpha\cos\phi}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi}\,, \qquad \mathcal{B}_{\tilde{9}\tilde{8}} = \frac{\frac{C}{r^4}\sin\phi\cos\phi\cosh^2\alpha}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi}\,,\\
e^{\Phi_b^{(10)}} &= \frac{1+\frac{C}{r^4}}{1+\frac{C}{r^4}\cosh^2\alpha\cos^2\phi}\,.
\end{aligned}
\tag{4.10}
$$

Once again, for the special case $\alpha = 0$, our solution reduces to the one in [13]. We have therefore presented a generalization of the $D1 - D3$ bound state to include new charges and momenta. The BPS nature of the new solution can be again examined by looking at the leading behavior of the gauge fields when the above solution is reduced along all its isometry directions: x^7, $\tilde{x}^8$, $\tilde{x}^9$. They are:

$$\begin{aligned} Q_1 &= 4C \cosh\alpha \sin\phi \,, \\ Q_2 &= -4C \cosh\alpha \cos\phi \,, \\ Q_3 &= -4C \sinh\alpha \cosh\alpha \cos\phi \,, \\ P &= -4C \sinh\alpha \cosh\alpha \sin\phi \,, \end{aligned} \tag{4.11}$$

where Q_1, Q_2, Q_3 and P are the charge associated with $\mathcal{A}^{(2)}_{7t}$, $\mathcal{A}^{(4)}_{\tilde{9}t\tilde{8}7}$, $\mathcal{B}_{\tilde{9}t}$ and $G_{t\tilde{8}}$, respectively.

In order to compute the ADM mass-density, we find:

$$\begin{aligned} h_{77} &= \frac{C}{r^4}\left(\frac{-3}{4} + \frac{1}{4}\cosh^2\alpha\cos^2\phi\right), \qquad & h_{\tilde{8}\tilde{8}} &= \frac{C}{r^4}\left(\cosh^2\alpha - \frac{3}{4}\cosh^2\alpha\cos^2\phi - \frac{3}{4}\right), \\ h_{\tilde{9}\tilde{9}} &= \frac{C}{r^4}\left(\frac{1}{4} - \frac{3}{4}\cosh^2\alpha\cos^2\phi\right), & h_{ij} &= \frac{C}{r^4}\left(\frac{1}{4} + \frac{1}{4}\cosh^2\alpha\cos^2\phi\right)\delta_{ij}\,, \end{aligned} \tag{4.12}$$

with h_{ij}'s being the deformations of the Einstein metric above flat background. One then gets, using (3.8), the mass-density of the $D1 - D3$ system as:

$$m_{1,3} = 4C\cosh^2\alpha\,. \tag{4.13}$$

We therefore once again have:

$$m_{1,3}^2 = Q_1^2 + Q_2^2 + Q_3^2 + P^2\,, \tag{4.14}$$

showing the BPS nature of the new bound state. Further generalization to higher $Dp - D(p+2)$ bound states can similarly be worked out. We therefore skip the details.

5. $\beta = 0, \alpha \neq 0$ solutions and generalization to (p,q)-strings

In this section, we discuss even more nontrivial examples, using $\beta = 0, \alpha \neq 0$ cases, discussed in section 2. We now also perform further generalization by applying T-duality on delocalized (p,q)-strings rather than $(0,1)$ or D-strings.

5.1 Construction of $\mathrm{SL}(2,\mathbb{Z})$ multiplets

The delocalized elementary string solution in ten-dimensions for $\beta = 0$, $\alpha \neq 0$ situation is given in eqs. (2.18)–(2.20). This solution represents a string along x^8, which is delocalized along x^9. One can use it to construct generalized bound states in the manner described in the last section. We, however, make further generalization by using the $\mathrm{SL}(2,\mathbb{Z})$ symmetry of type-IIB string theories in $D = 10$. We can easily construct a (p,q) multiplet of delocalized string from (2.18)–(2.20). The procedure of constructing such configuration is discussed in [19]. Instead of giving the detail, we write down the final form of the configuration.

The metric is given by:

$$ds^2 = \frac{A}{\Delta}\Big(- \Big[1 - \sinh^2\frac{\alpha}{2}\left(e^{-E}-1\right)\Big]\, dt^2 + \Big[1 + \sinh^2\frac{\alpha}{2}\left(e^{-E}-1\right)\Big]\left(dx^8\right)^2 + \\ + \Delta\left(dx^9\right)^2 + \sinh\alpha\left(e^{-E}-1\right) dx^9 dt + 2\sinh^2\frac{\alpha}{2}\left(e^{-E}-1\right) dt dx^8 + \\ + \sinh\alpha\left(e^{-E}-1\right) dx^8 dx^9 + \Delta\sum_{i=1}^{7} dx^i dx^i\Big)\,, \tag{5.1}$$

where we have defined

$$\Delta = \cosh^2\frac{\alpha}{2} e^{-E} - \sinh^2\frac{\alpha}{2}\,, \qquad \text{and} \qquad A = \frac{\sqrt{p^2+q^2\Delta}}{\sqrt{p^2+q^2}}\,. \tag{5.2}$$

Furthermore, the NS-NS and R-R two forms are given by

$$\begin{aligned} B^{(1)}_{8t} &= \frac{p}{\sqrt{p^2+q^2}} B_{8t}\,, \qquad & B^{(1)}_{9t} &= \frac{p}{\sqrt{p^2+q^2}} B_{9t} = B^{(1)}_{98}\,, \\ B^{(2)}_{8t} &= \frac{q}{\sqrt{p^2+q^2}} B_{8t}\,, \qquad & B^{(2)}_{9t} &= \frac{q}{\sqrt{p^2+q^2}} B_{9t} = B^{(2)}_{98}\,. \end{aligned} \tag{5.3}$$

Here B_{8t}, B_{9t}, B_{98} are given by (2.19). The superscripts 1 and 2 indicate that these two forms are NS-NS and R-R in nature respectively. Also, after $\mathrm{SL}(2,\mathbb{Z})$ transformation, we have the dilaton and axion as:

$$\Phi^{(10)}_b = -2\ln\left(\frac{(p^2+q^2)\Delta^{\frac{1}{2}}}{p^2+q^2\Delta}\right), \qquad \chi = \frac{pq(\Delta-1)}{p^2+q^2\Delta}\,. \tag{5.4}$$

5.2 Construction of bound states

Now, we perform, as before, a rotation on the above (p,q) string solution in the $x^9 - x^8$ plane. This is again given by

$$\begin{aligned} dx^9 &= \cos\phi\; d\tilde{x}^9 - \sin\phi\; d\tilde{x}^8\,, \\ dx^8 &= \cos\phi\; d\tilde{x}^8 + \sin\phi\; d\tilde{x}^9\,. \end{aligned} \tag{5.5}$$

Under this rotation, (5.1)–(5.3) become:

$$ds^2 = \frac{A}{\Delta}\Big(-(1-\delta_\alpha)dt^2 + \left[\sin^2\phi(1+\delta_\alpha) + \gamma_\alpha\sin\phi\cos\phi + \Delta\cos^2\phi\right]\left(d\tilde{x}^9\right)^2 + \\ + \left[\cos^2\phi(1+\delta_\alpha) - \gamma_\alpha\sin\phi\cos\phi + \Delta\sin^2\phi\right]\left(d\tilde{x}^8\right)^2 + \\ + \left[\gamma_\alpha\cos 2\phi + 2(\delta_\alpha - \Delta + 1)\cos\phi\sin\phi\right] d\tilde{x}^9 d\tilde{x}^8 + \left[2\delta_\alpha\sin\phi + \gamma_\alpha\cos\phi\right] d\tilde{x}^9 dt + \\ + \left[2\delta_\alpha\cos\phi - \gamma_\alpha\sin\phi\right] d\tilde{x}^8 dt + \Delta\sum_{i=1}^{7}\left(dx^i\right)^2\Big)\,, \tag{5.6}$$

and

$$
\begin{aligned}
B^{(1)}_{\tilde{8}t} &= \frac{p}{\Delta\sqrt{p^2+q^2}}\left[\cos\phi\left(e^{-E}-1+\delta_\alpha\right)+\frac{\sin\phi\gamma_\alpha}{2}\right],\\
B^{(1)}_{\tilde{9}t} &= \frac{p}{\Delta\sqrt{p^2+q^2}}\left[\sin\phi\left(e^{-E}-1+\delta_\alpha\right)-\frac{\cos\phi\gamma_\alpha}{2}\right],\\
B^{(1)}_{\tilde{9}\tilde{8}} &= -\frac{p\gamma_\alpha}{2\Delta\sqrt{p^2+q^2}},\\
B^{(2)}_{\tilde{8}t} &= \frac{q}{\Delta\sqrt{p^2+q^2}}\left[\cos\phi\left(e^{-E}-1+\delta_\alpha\right)+\frac{\sin\phi\gamma_\alpha}{2}\right],\\
B^{(2)}_{\tilde{9}t} &= \frac{q}{\Delta\sqrt{p^2+q^2}}\left[\sin\phi\left(e^{-E}-1+\delta_\alpha\right)-\frac{\cos\phi\gamma_\alpha}{2}\right],\\
B^{(2)}_{\tilde{9}\tilde{8}} &= -\frac{q\gamma_\alpha}{2\Delta\sqrt{p^2+q^2}}.
\end{aligned}
\tag{5.7}
$$

Here, we have defined

$$
\delta_\alpha = \sinh^2\frac{\alpha}{2}\left(e^{-E}-1\right), \qquad \gamma_\alpha = \sinh\alpha\left(e^{-E}-1\right), \tag{5.8}
$$

Furthermore, the dilaton and axion remain same as (5.4) .

Our final step of the construction is to apply T-duality along $\tilde{x}^9$ direction. The resulting solution would then correspond to a bound state in type-IIA theory. The T-duality map between IIA and IIB theories is given explicitly in [17, 13]. Applying this map, we end up with resulting configuration:

$$
\begin{aligned}
d\mathcal{S}^2 = {} & \frac{1}{\Delta\sigma\sqrt{p^2+q^2}\sqrt{p^2+q^2\Delta}}\times\\
& \times\Big[p^2\left\{\sin\phi\left(e^{-E}+\delta_\alpha\right)\left(\sin\phi\left(e^{-E}-2+\delta_\alpha\right)-\cos\phi\gamma_\alpha\right)-\Delta\cos^2\phi(1-\delta_\alpha)\right\}-\\
& \quad -q^2\Delta\left\{\left(\sin\phi+\frac{\gamma_\alpha}{2}\cos\phi\right)^2+\Delta\cos^2\phi(1-\delta_\alpha)\right\}\Big](dt)^2+\\
& +\frac{1}{2\Delta\sigma\sqrt{p^2+q^2}\sqrt{p^2+q^2\Delta}}\times\\
& \times\left[\Delta\left\{-\gamma_\alpha^2q^2+4\delta_\alpha\left(p^2+\Delta q^2\right)\right\}\cos\phi-2\gamma_\alpha\left\{\delta_\alpha p^2+e^{-E}p^2+\Delta q^2\right\}\sin\phi\right]dt\,d\tilde{x}^8+\\
& +\frac{\Delta\sqrt{p^2+q^2}}{\sigma\sqrt{p^2+q^2\Delta}}d\tilde{x}^9d\tilde{x}^9-\frac{p}{\sigma\sqrt{p^2+q^2\Delta}}\left[2\sin\phi\left(e^{-E}-1+\delta_\alpha\right)-\gamma_\alpha\cos\phi\right]d\tilde{x}^9dt+\\
& +\frac{1}{\Delta\sigma\sqrt{p^2+q^2\Delta}\sqrt{p^2+q^2}}\times\\
& \times\Big[\left(\Delta+\delta_\alpha\Delta-\frac{\gamma_\alpha^2}{4}\right)\left(p^2+q^2\Delta\right)+\\
& \quad +p^2\left\{\frac{-\gamma_\alpha}{2}\cos\phi+\left(e^{-E}-1+\delta_\alpha\right)\sin\phi\right\}^2\Big]d\tilde{x}^8d\tilde{x}^8+\\
& +\frac{p\gamma_\alpha}{\sigma\sqrt{p^2+q^2\Delta}}d\tilde{x}^8d\tilde{x}^9+\frac{\sqrt{p^2+q^2\Delta}}{\sqrt{p^2+q^2}}\sum_{i=1}^{7}dx^idx^i.
\end{aligned}
\tag{5.9}
$$

The IIA dilaton is

$$e^{\Phi_a^{(10)}} = \left(\frac{p^2+q^2\Delta}{p^2+q^2}\right)^{\frac{3}{2}} \frac{1}{\sigma}\,. \tag{5.10}$$

The two form components are given by

$$\begin{aligned}
\mathcal{B}_{t\tilde{8}} &= -\frac{p}{2\Delta\sigma\sqrt{p^2+q^2}}\left[-\gamma_\alpha\left(e^{-E}+\delta_\alpha\right)\sin\phi + 2\Delta\left(-e^{-E}+1-\delta_\alpha\right)\cos\phi\right],\\
\mathcal{B}_{t\tilde{9}} &= \frac{1}{\sigma}\left[\delta_\alpha\sin\phi + \frac{\gamma_\alpha}{2}\cos\phi\right],\\
\mathcal{B}_{\tilde{8}\tilde{9}} &= \frac{1}{\sigma}\left[\frac{\gamma_\alpha}{2}\cos 2\phi + (\delta_\alpha-\Delta+1)\cos\phi\sin\phi\right].
\end{aligned} \tag{5.11}$$

The nonzero components of 1-form and 3-form fields in IIA theory are given by

$$\begin{aligned}
\mathcal{A}_t &= \frac{q\sqrt{p^2+q^2}}{p^2+q^2\Delta}\left[\sin\phi\left(e^{-E}-1+\delta_\alpha\right) - \gamma_\alpha\frac{\cos\phi}{2}\right],\\
\mathcal{A}_{\tilde{8}} &= -\frac{q\gamma_\alpha\sqrt{p^2+q^2}}{2\left(p^2+\Delta q^2\right)}\\
\mathcal{A}_{\tilde{9}} &= -\frac{pq(\Delta-1)}{p^2+q^2\Delta}\\
\mathcal{A}_{t\tilde{8}\tilde{9}} &= \frac{q}{2\sigma\Delta\sqrt{p^2+q^2}}\left[-\gamma_\alpha\left(e^{-E}+\delta_\alpha\right)\sin\phi + 2\Delta\left(-e^{-E}+1-\delta_\alpha\right)\cos\phi\right],
\end{aligned} \tag{5.12}$$

where

$$\sigma = \sin^2\phi(1+\delta_\alpha) + \gamma_\alpha\sin\phi\cos\phi + \Delta\cos^2\phi\,, \tag{5.13}$$

and

$$e^{-E} = 1 + \frac{C}{r^5}\,. \tag{5.14}$$

We, once again, have multiple nonzero components of the NS-NS 2-form as well as R-R 1-form and 3-form fields, in addition to having nonzero momenta coming from the off-diagonal components of the metric. We have explicitly checked that the above configuration reduces to known D-brane bound states in appropriate limits.

5.3 Mass-charge relationship

As in section 4.2, we now verify that mass and charge associated with the above solution satisfy the expected mass-charge relation of 1/2 (non-threshold) BPS bound states. As in section 4.2, we once again reduce the solution along both its spatial isometry directions. In the resulting theory in $D=8$, nonzero charges are associated with gauge fields from the IIA field reductions. We have the non zero gauge fields which are of electric type, given as: $A_t^1 = \mathcal{A}_t$, $A_t^2 = \mathcal{A}_{t\tilde{8}\tilde{9}}$, $A_t^3 = -\mathcal{B}_{t\tilde{8}}$, $A_t^4 = \mathcal{B}_{t\tilde{9}}$, $A_t^5 = g_{t\tilde{8}}/g_{\tilde{8}\tilde{8}}$, $A_t^6 = g_{t\tilde{9}}/g_{\tilde{9}\tilde{9}}$ [20]. The charges can be read off from the leading order behavior of the above expressions, and are as follows:

$$
\begin{aligned}
Q_1 &= \frac{5Cq}{\sqrt{p^2+q^2}} \times \cosh\frac{\alpha}{2}\left(\sin\phi\cosh\frac{\alpha}{2} - \cos\phi\sinh\frac{\alpha}{2}\right),\\
Q_2 &= -\frac{5Cq}{\sqrt{p^2+q^2}} \times \cosh\frac{\alpha}{2}\left(\cos\phi\cosh\frac{\alpha}{2} + \sin\phi\sinh\frac{\alpha}{2}\right),\\
Q_3 &= -\frac{5Cp}{\sqrt{p^2+q^2}} \times \cosh\frac{\alpha}{2}\left(\cos\phi\cosh\frac{\alpha}{2} + \sin\phi\sinh\frac{\alpha}{2}\right),\\
Q_4 &= 5C \times \sinh\frac{\alpha}{2}\left(\sin\phi\sinh\frac{\alpha}{2} + \cos\phi\cosh\frac{\alpha}{2}\right),\\
P_1 &= 5C \times \sinh\frac{\alpha}{2}\left(\cos\phi\sinh\frac{\alpha}{2} - \sin\phi\cosh\frac{\alpha}{2}\right),\\
P_2 &= \frac{5Cp}{\sqrt{p^2+q^2}} \times \cosh\frac{\alpha}{2}\left(\sin\phi\cosh\frac{\alpha}{2} - \cos\phi\sinh\frac{\alpha}{2}\right),
\end{aligned}
\tag{5.15}
$$

where Q_i corresponds to the charge associated with A_t^i, $(i = 1,..,4)$ and the P_i, $(i = 1, 2)$ correspond to the charges with respect to the gauge fields A_t^5 and A_t^6. They also correspond to the momenta along $\tilde{x}^8$ and $\tilde{x}^9$ in the original uncompactified theory. The ADM mass density of the bound state, that we have constructed is

$$
m_{(0,2)}^{(p,q)} = 5C\cosh\alpha\,. \tag{5.16}
$$

Therefore we have

$$
\left(m_{(0,2)}^{(p,q)}\right)^2 = {Q_1}^2 + {Q_2}^2 + {Q_3}^2 + {Q_4}^2 + {P_1}^2 + {P_2}^2\,. \tag{5.17}
$$

This relation implies that the bound state constructed above do satisfy the BPS bound for the system. As in section 4, one can also give a generalization of $Dp - D(p+2)$ bound states in a similar manner, as discussed above, by smearing directions x^7 etc. in the solution (5.9)–(5.12) and then by applying T-duality along these directions. We, however, skip these details.

It is interesting to note that we have been able to generate all six gauge charges in $D = 8$, using our procedure. As is known, these gauge charges form a $(3, 2)$ representation of $\mathrm{SL}(3,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$ U-duality symmetry in $D = 8$. We can therefore rewrite the RHS of equation (5.17) in a U-duality invariant form by exciting general moduli as in [19, 20]. This is not surprising, as solution generating technique used by us is known to be equivalent to the one using U-duality transformations [21, 22, 23]. In particular, in examples where we consider particle-like states in $D = 8$ for writing down the mass-charge relations, the relevant U-duality group is $\mathrm{SL}(3,\mathbb{Z})\times\mathrm{SL}(2,\mathbb{Z})$. In type-IIB examples, this $\mathrm{SL}(2,\mathbb{Z})$ in $D = 8$ originates from the group of constant coordinate transformations along the two internal directions, whereas $\mathrm{SL}(3,\mathbb{Z})$ is a combination of the $D = 10$ S-duality group together with the T-duality along the x^8 and x^9. $(3, 2)$ multiplet of states mentioned above are then generated by applying these transformations to a seed solution in $D = 8$, orginating from the (delocalized) F-string solution in $D = 10$. All our transformations can also be mapped to appropriate ones lying inside the U-duality group.

JHEP05(2002)050

6. Conclusion

In this paper, we have explicitly constructed nontrivial bound states of D- branes starting with charged macroscopic strings. In particular, we were able to construct configurations in ten dimensions which carry $(F, D0, D2)$ charges as well as non-zero momenta. We also found that these bound states are supersymmetric. We checked that all our solutions reduce to known bound states in appropriate limits. We then further generalized our results to $(Dp - D(p+2))$ bound states in IIA/B theories.

The bound states of D-branes, when compactified to lower dimensions, often allow us to understand various properties of black hole including Bekenstein-Hawking entropy in a microscopic way [24, 25, 26]. It would be interesting to investigate as to what new insight we gain from our configurations along this direction. A first step may then be to reduce the solutions that we presented in sections 3 and 4.1 to $D = 8$ and $D = 7$ respectively to interpret them as charged extremal black holes in lower dimensions. Various excitations of the higher dimensional bound states along the compactified direction may then correspond to the required degrees of freedom responsible for the entropy associated with the black hole. We hope to persue along this direction in future. It would also, perhaps, be interesting to understand the conformal field theory descriptions of these bound states. They are typically described by boundary states of the underlying open string theory. Construction of these boundary states often turned out to be very useful in the past, see for example [27, 28].

References

[1] E. Witten, *Bound states of strings and p-branes*, *Nucl. Phys.* **B 460** (1996) 335 [hep-th/9510135].

[2] M. Li, *Boundary states of D-branes and Dy-strings*, *Nucl. Phys.* **B 460** (1996) 351 [hep-th/9510161].

[3] M.R. Douglas, *Branes within branes*, hep-th/9512077.

[4] M.B. Green, N.D. Lambert, G. Papadopoulos and P.K. Townsend, *Dyonic p-branes from self-dual (p + 1)-branes*, *Phys. Lett.* **B 384** (1996) 86 [hep-th/9605146].

[5] J.G. Russo and A.A. Tseytlin, *Waves, boosted branes and BPS states in M-theory*, *Nucl. Phys.* **B 490** (1997) 121 [hep-th/9611047].

[6] M.S. Costa and G. Papadopoulos, *Superstring dualities and p-brane bound states*, *Nucl. Phys.* **B 510** (1998) 217 [hep-th/9612204].

[7] H. Arfaei and M.M. Sheikh Jabbari, *Mixed boundary conditions and brane, string bound states*, *Nucl. Phys.* **B 526** (1998) 278 [hep-th/9709054].

[8] M.M. Sheikh-Jabbari, *More on mixed boundary conditions and D-branes bound states*, *Phys. Lett.* **B 425** (1998) 48 [hep-th/9712199].

[9] J.M. Maldacena and J.G. Russo, *Large-N limit of non-commutative gauge theories*, *J. High Energy Phys.* **09** (1999) 025 [hep-th/9908134].

[10] A. Hashimoto and K. Hashimoto, *Monopoles and dyons in non-commutative geometry*, *J. High Energy Phys.* **11** (1999) 005 [hep-th/9909202].

[11] M. Alishahiha, Y. Oz and M.M. Sheikh-Jabbari, *Supergravity and large-N noncommutative field theories*, *J. High Energy Phys.* **11** (1999) 007 [hep-th/9909215].

[12] A. Sen, *Macroscopic charged heterotic string*, *Nucl. Phys.* **B 388** (1992) 457 [hep-th/9206016].

[13] J.C. Breckenridge, G. Michaud and R.C. Myers, *More D-brane bound states*, *Phys. Rev.* **D 55** (1997) 6438 [hep-th/9611174].

[14] A. Kumar, *Charged macroscopic type-II strings and their networks*, *J. High Energy Phys.* **12** (1999) 001 [hep-th/9911090].

[15] A. Sen, *String string duality conjecture in six-dimensions and charged solitonic strings*, *Nucl. Phys.* **B 450** (1995) 103 [hep-th/9504027].

[16] A. Sen, *Strong-weak coupling duality in four-dimensional string theory*, *Int. J. Mod. Phys.* **A 9** (1994) 3707 [hep-th/9402002].

[17] E. Bergshoeff, C.M. Hull and T. Ortin, *Duality in the type-II superstring effective action*, *Nucl. Phys.* **B 451** (1995) 547 [hep-th/9504081].

[18] J.X. Lu, *Adm masses for black strings and p-branes*, *Phys. Lett.* **B 313** (1993) 29 [hep-th/9304159].

[19] J.H. Schwarz, *An* $\mathrm{SL}(2,\mathbb{Z})$ *multiplet of type-IIB superstrings*, *Phys. Lett.* **B 360** (1995) 13 [hep-th/9508143].

[20] J.X. Lu and S. Roy, *U-duality p-branes in diverse dimensions*, *Nucl. Phys.* **B 538** (1999) 149 [hep-th/9805180].

[21] C.M. Hull and P.K. Townsend, *Unity of superstring dualities*, *Nucl. Phys.* **B 438** (1995) 109 [hep-th/9410167].

[22] J.M. Izquierdo, N.D. Lambert, G. Papadopoulos and P.K. Townsend, *Dyonic membranes*, *Nucl. Phys.* **B 460** (1996) 560 [hep-th/9508177].

[23] M.S. Costa and M. Cvetič, *Non-threshold D-brane bound states and black holes with non-zero entropy*, *Phys. Rev.* **D 56** (1997) 4834 [hep-th/9703204].

[24] A. Sen, *Extremal black holes and elementary string states*, *Mod. Phys. Lett.* **A 10** (1995) 2081 [hep-th/9504147].

[25] A. Hindawi, B.A. Ovrut and D. Waldram, *Two-dimensional higher-derivative supergravity and a new mechanism for supersymmetry breaking*, *Nucl. Phys.* **B 471** (1996) 409 [hep-th/9509174].

[26] C.G. Callan and J.M. Maldacena, *D-brane approach to black hole quantum mechanics*, *Nucl. Phys.* **B 472** (1996) 591 [hep-th/9602043].

[27] P. Di Vecchia and A. Liccardo, *D-branes in string theory 2*, hep-th/9912275.

[28] P. Di Vecchia, M. Frau, A. Lerda and A. Liccardo, *(F,Dp) bound states from the boundary state*, *Nucl. Phys.* **B 565** (2000) 397 [hep-th/9906214].